

# Atomic Quantum Zeno Effect for Ensembles and Single Systems

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## Abstract

The so-called quantum Zeno effect is essentially a consequence of the projection postulate for ideal measurements. To test the effect Itano et al. have performed an experiment on an ensemble of atoms where rapidly repeated level measurements were realized by means of short laser pulses. Using dynamical considerations we give an explanation why the projection postulate can be applied in good approximation to such measurements. Corrections to ideal measurements are determined explicitly. This is used to discuss in how far the experiment of Itano et al. can be considered as a test of the quantum Zeno effect. We also analyze a new possible experiment on a single atom where stochastic light and dark periods can be interpreted as manifestation of the quantum Zeno effect. We show that the measurement point of view gives a quick and intuitive understanding of experiments of the above type, although a finer analysis has to take the corrections into account.

## 1. Introduction

For an ideal measurement of an observable  $A$  on a system in state  $|\psi\rangle$  standard quantum mechanics predicts as possible outcomes the eigenvalues  $a_i$  of  $A$ . Each  $a_i$  is found with probability  $\|\mathbb{P}_i|\psi\rangle\|^2$ , where  $\mathbb{P}_i$  is the projector onto the eigenspace belonging to  $a_i$ . The projection postulate then states that right after a measurement for which  $a_i$  is found the system is in the state  $\mathbb{P}_i|\psi\rangle$ . The projection postulate as currently used has been formulated by Lüders [1]. For observables with degenerate eigenvalues his formulation differs from that of von Neumann [2]. Lüders stressed its provisional character: “The projection postulate will be employed only until a better understanding of the actual measurement process has been found” [3]. As pointed out to us by Sudbury [4], in the first edition of his famous book Dirac [5] defines observations causing minimal disturbance.

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These correspond to Lüder's prescription. Curiously though, in later editions this passage has been omitted.

One consequence of the projection postulate is, under some mild technical conditions, the so-called quantum Zeno effect [6]. It predicts a slow-down of the time development due to rapidly repeated measurements. If the time between two measurements,  $\Delta t$ , goes to zero the system is frozen on a subspace.

An experiment to test the quantum Zeno effect was performed by Itano et al. [7]. They stored several thousand two-level ions in a trap. Initially all ions were prepared in the ground state. Then a pulse of a weak field in resonance was applied which pumped an ion in the ground state  $|1\rangle$  to the excited state  $|2\rangle$ . Such a pulse is called a  $\pi$  pulse. During this  $\pi$  pulse  $n$  population measurements of the ion levels were performed. Starting in level 1 and increasing the number  $n$  of measurements, i.e. decreasing the time  $\Delta t$  between two measurements, less atoms were found in level 2 at the end of the  $\pi$  pulse. For  $n = 1$  to 64 the results were in good agreement with the predictions of the quantum Zeno effect.

Opinions have been divided in the literature about the relevance of this experiment for the quantum Zeno effect (cf. e.g. Refs. [8, 9, 10, 11, 12, 13, 14, 15]). Some have called it a dramatic verification of the effect. Other maintained that it had nothing to do with the effect since the measurements were realized by short laser pulses which should be included in the dynamics. Therefore the experiment can be understood either by Bloch equations [11, 12, 14, 15], which describe the interaction of an ensemble of atoms with the laser, or by incorporating the laser pulses as an external field in the Hamiltonian [9]. Thus one can describe the effect of the laser pulses in a purely dynamical way without any measurement interpretation.

Namiki and collaborators have investigated the quantum Zeno effect and possible experimental verifications for other systems, in particular spin systems, and have discussed the connection with measurement theory [16]. The quantum Zeno effect and the experiment of Ref. [7] have given rise to a large number of publications [17].

The aim of this paper is twofold. First, we want to discuss the slightly different roles the quantum Zeno effect plays for an ensemble as opposed to a single system. Second, drawing on recent results of ours [18, 19, 20] we will describe a state measurement on the two-level atoms by short laser pulses in a way which, although also dynamical, shows why the laser pulses are so well described by the notion of measurement. We will explain how and why the measurement point of view gives such a quick and intuitive understanding of experiments of the above type, especially of experiments with *single* systems. However, the description by ideal measurements is only approximate. Corrections to ideal measurements are

necessary and can be determined explicitly.

## 2. Role of the projection postulate for single systems and ensembles

In principle the projection postulate deals with ideal measurements on individual systems (selective measurements). E.g. for a single system in state  $|\psi\rangle$  one can measure  $|\psi\rangle$ , i.e. the observable  $|\psi\rangle\langle\psi|$ , at times  $\Delta t, 2\Delta t, \dots, n\Delta t = t$ . Then the probability to find the system at each measurement in  $|\psi\rangle$  until  $t$  equals

$$P(t) = \|U(\Delta t)|\psi\rangle\langle\psi| \dots |\psi\rangle\langle\psi| U(\Delta t)|\psi\rangle\langle\psi| U(\Delta t)|\psi\rangle\|^2 \quad (1)$$

$$= 1 + \Delta t \frac{t}{\hbar} \left( \langle\psi|H|\psi\rangle^2 - \langle\psi|H^2|\psi\rangle \right) + \mathcal{O}(\Delta t^2) . \quad (2)$$

Here the Hamiltonian is taken to be constant in time and suitable domain assumptions have been made. In the limit when the time  $\Delta t$  between two measurements goes to zero one has

$$\lim_{\Delta t \rightarrow 0} P(t) = 1 . \quad (3)$$

In this (idealized) limit the system freezes in its initial state. For a system described by a finite dimensional Hilbert space the domain assumptions are always fulfilled. The general case can be treated with less stringent technical assumptions [6].

In an ensemble of systems one may initially prepare all systems in the same state  $|\psi\rangle$ . Then the density matrix  $\rho(t)$  after  $n$  measurements (non-selective measurements) is a mixture ('incoherent superposition') of various subensembles resulting from selective measurements on individual systems. Now  $P(t)$ , as determined in Eq. (2), is equal to the relative size of the subensemble found in  $|\psi\rangle$  at each measurement. Eq. (3) shows that for decreasing  $\Delta t$  this subensemble increases and thus the density matrix remains closer to the initial state for longer times, i.e. there is an overall slow-down of the time-development of the density matrix and eventually a freezing.

As an example we consider ideal state measurements on a two-level system [21, 22]. Both levels are assumed to be stable. The transition between level 1 and 2 is driven by a weak field in resonance. This leads to so-called Rabi oscillations in which the atomic state oscillates continuously between  $|1\rangle$  and  $|2\rangle$ . In an appropriate interaction picture the time development operator reads

$$U(t) = \cos \frac{\Omega_2}{2} t - i \sin \frac{\Omega_2}{2} t (|1\rangle\langle 2| + |2\rangle\langle 1|) , \quad (4)$$

where the so-called Rabi frequency  $\Omega_2$  is proportional to the amplitude of the driving field. Now one can perform rapidly repeated ideal measurements of  $|1\rangle\langle 1|$  or  $|2\rangle\langle 2|$  at times  $\Delta t$  apart. Then the probability to find another measurement

result than before is given by

$$P(|1\rangle \rightarrow |2\rangle) = P(|2\rangle \rightarrow |1\rangle) = \sin^2 \frac{\Omega_2}{2} \Delta t , \quad (5)$$

which is proportional to  $\Delta t^2$  if  $\Omega_2 \Delta t \ll 1$ . For  $|\psi\rangle = |1\rangle$  or  $|2\rangle$ , Eq. (1) becomes

$$P(t) = \cos^{2n} \frac{\Omega_2}{2} \Delta t . \quad (6)$$

This leads to a stochastic path for a single atom as shown in Fig. 1. Each ideal measurement projects the atom onto state  $|1\rangle$  or  $|2\rangle$ , respectively. Then the increase or decrease of the population of level 2 goes initially quadratic in time. As can be seen in Fig. 1, more frequent measurements lead to a slow-down of the time-development of the atomic state.

Let us imagine that the measurement apparatus emits a light signal each time the atom is found in  $|1\rangle$ . Then one would observe stochastically alternating light and dark periods. In Fig. 2 the lines mark times when the atom is found to be in state  $|1\rangle$ , with the accompanying light signal. When the time between two measurements becomes smaller, the periods where the atom is found without interruptions to be in  $|1\rangle$  becomes longer. This corresponds to a light period. The same holds for dark periods, with no light signals. The atom seems to stay in one state for some length of time. As discussed further below this suggests a possible experimental demonstration of the quantum Zeno effect for a single atom.

From Eq. (5) the mean length and the standard deviation of the light and dark periods can be obtained. One finds, with  $t_n = n\Delta t$ ,

$$\overline{T}_L = \sum_{n=1}^{\infty} n\Delta t [P(t_{n-1}) - P(t_n)] \quad (7)$$

and the same expression for  $\overline{T}_D$ . Thus, with Eq. (6), one easily obtains

$$\overline{T}_L = \overline{T}_D = \frac{\Delta t}{\sin^2 \frac{1}{2} \Omega_2 \Delta t} . \quad (8)$$

The standard deviation is found as

$$\Delta T_L = \Delta T_D = \Delta t \frac{\cos \frac{1}{2} \Omega_2 \Delta t}{\sin^2 \frac{1}{2} \Omega_2 \Delta t} . \quad (9)$$

For small  $\Delta t$  one has a very broad distribution (see Fig. 3). In this case of ideal measurements there is a symmetry between light and dark periods.

When measuring repeatedly on an ensemble (“gas”) of atoms and assuming a light signal for each individual atom found in  $|1\rangle$  would have a statistical overlap of individual light and dark periods, and the individual periods would be no

longer discernible. One would just have a decrease of the overall luminosity, from which one could deduce the above mentioned slow-down in the time development of the density matrix.

### 3. Analysis of a measurement proposal

Cook [21] made a proposal how to measure whether the atom is in state  $|1\rangle$  or  $|2\rangle$ . His idea was to use an auxiliary, rapidly decaying third level and a short strong laser pulse. Occurrence and absence of fluorescence was taken as indication that the atom was found in state  $|1\rangle$  and  $|2\rangle$ , respectively. That fluorescence yields the state  $|1\rangle$  is quite easy to understand. Let us first assume that, as shown in Fig. 4, the 1–2 transition is not driven (no  $\pi$  pulse) when the strong laser pulse is applied. Then one can argue as follows. If the laser pulse produces fluorescence then after the last photon emitted during the laser pulse the atom is in the ground state. Until the end of the laser pulse the atom is again driven into a superposition of level 1 and 3. Then the  $|3\rangle$  component decays in a short transient time  $\tau_{\text{tr}}$ . Hence shortly after the end of the laser pulse the atom is in state  $|1\rangle$ . If the atom is initially in state  $|2\rangle$  no photons are emitted, because a transition to level 3 is not possible. Thus for the total system (atom plus radiation field) the time development until after the end of the laser pulse should transform

$$\begin{array}{lll} |1\rangle |\text{vacuum}\rangle & \text{into} & |1\rangle |\text{photons}\rangle \\ |2\rangle |\text{vacuum}\rangle & \text{into} & |2\rangle |\text{vacuum}\rangle \end{array}$$

where the state  $|\text{photons}\rangle$  contains practically no vacuum part. By linearity one then has for an arbitrary initial state the transformation

$$(\alpha_1|1\rangle + \alpha_2|2\rangle) |\text{vacuum}\rangle \longmapsto \alpha_1|1\rangle |\text{photons}\rangle + \alpha_2|2\rangle |\text{vacuum}\rangle .$$

Complications arise if the 1–2 transition is driven. In how far Cooks proposal [21] corresponds to an ideal measurement and state reduction can be analyzed in detail by means of the quantum jump approach [23, 24, 25, 26], which is essentially equivalent to quantum trajectories [27] and to the Monte-Carlo wave function approach [28].

With the quantum jump approach one can show that an atom evolves with a so-called conditional time development operator  $U_{\text{cond}}$  as long as no photons are detected. Thus an atom, at time  $t_0$  in the initial state

$$|\psi\rangle = \alpha_1|1\rangle + \alpha_2|2\rangle , \tag{10}$$

is at time  $t$  in the state

$$|\psi^0(t)\rangle = U_{\text{cond}}(t, t_0)|\psi\rangle \equiv \exp\left(-\frac{i}{\hbar}(t - t_0)H_{\text{cond}}\right)|\psi\rangle , \tag{11}$$

if no photons are found between  $t_0$  and  $t$ . The conditional Hamiltonian  $H_{\text{cond}}$  can be explicitly calculated. In the case of a system as in Fig. 5 one has, in the same interaction picture as in Eq. (4),

$$H_{\text{cond}} = \frac{\hbar}{2} \left[ \sum_{i=2}^3 \Omega_i (|1\rangle\langle i| + |i\rangle\langle 1|) - iA_3|3\rangle\langle 3| \right] . \quad (12)$$

The conditional Hamiltonian is non-Hermitian and the norm of  $|\psi^0(t)\rangle$  decreases in time. The probability to find no photon between  $t_0$  and  $t$  is given by

$$P_0(t, \psi) = \|\psi^0(t)\|^2 = \|U_{\text{cond}}(t, t_0)|\psi\rangle\|^2 .$$

The probability density for the emission of the first photon in the interval  $[t, t+dt]$  is equal to  $w_1(t, \psi)dt$  with

$$w_1(t, \psi) = -\frac{d}{dt}P_0(t, \psi) .$$

It is also shown in the quantum jump approach that a system is in state  $|1\rangle$  after the emission of a photon if its level structure is as in Figs. 4 and 5.

This can now be used to analyze the effect of the laser pulse on an atom in more detail. If a laser pulse produces no fluorescence, the state of the atom at the end of the laser pulse is given by

$$|\psi^0(\tau_p)\rangle = \exp\left(-\frac{i}{\hbar}H_{\text{cond}}\tau_p\right)|\psi\rangle .$$

If we consider first the case where no field is applied to the 1–2 transition ( $\Omega_2 = 0$ ), the eigenvalues  $\lambda_i$ ,  $i = 1, 2, 3$ , of  $H_{\text{cond}}$  are easily calculated. One of them, denoted by  $\lambda_2$ , is zero and the corresponding eigenvector is  $|2\rangle$ . The two remaining eigenvalues have negative imaginary part. If all eigenvalues are pairwise different (otherwise one has to take limits) one can write

$$|\psi^0(\tau_p)\rangle = \sum_{i=1}^3 \exp\left(-\frac{i}{\hbar}\lambda_i\tau_p\right) |\lambda_i\rangle\langle\lambda^i|\psi\rangle , \quad (13)$$

where the  $|\lambda_i\rangle$ 's are the eigenvectors of the conditional Hamiltonian and the  $\langle\lambda^i|$ 's the reciprocal vectors, with  $\langle\lambda^j|\lambda_i\rangle = \delta_{ij}$ . If  $\tau_p$  is large enough the exponentials with  $i = 1$  and  $i = 3$  in Eq. (13) have dropped off to zero and thus at the end of the laser pulse the state of an atom without any emission has become

$$|\psi^0(\tau_p)\rangle = \alpha_2|2\rangle . \quad (14)$$

As shown in Refs. [18, 19] this is valid if the duration of the laser pulse satisfies the condition

$$\tau_p \gg \max\left\{1/A_3, A_3/\Omega_3^2\right\} . \quad (15)$$

Then the probability to find no photon equals  $|\alpha_2|^2$ , and the atom is in  $|2\rangle$  at the end of the laser pulse. Analogously a condition on the transient time  $\tau_{\text{tr}}$  can be determined. The transient time has to be long enough to allow the vanishing of possible  $|3\rangle$  components,

$$\tau_{\text{tr}} \gg 1/A_3 . \quad (16)$$

Summarizing the discussion so far, if  $\Omega_2 = 0$  the effect of the laser pulse can be interpreted as a projection of the atom onto states  $|1\rangle$  or  $|2\rangle$ , respectively, if conditions (15) and (16) are satisfied. This happens with the same probabilities as predicted for an ideal measurement. The resulting state is characterized by occurrence, or no occurrence, of a burst of light. The time this takes is equal to  $\tau_p + \tau_{\text{tr}}$ , the sum of laser pulse length and the transient time.

If the weak field is not turned off when the laser pulse is on, as shown in Fig. 5, some complications arise. The weak field can cause a small additional transition between levels 1 and 2 while the strong laser pulse is on, as well as during the transient time. This leads to small corrections, which we have explicitly determined in Refs. [18, 19] by means of the quantum jump approach. One has three parameters which have to be small to make the following interpretation possible, namely

$$\epsilon_A = \frac{\Omega_2}{A_3} \ll 1, \quad \epsilon_R = \frac{\Omega_2}{\Omega_3} \ll 1 \quad \text{and} \quad \epsilon_p = \frac{\Omega_2 A_3}{\Omega_3^2} \ll 1 . \quad (17)$$

Now one can say that if the condition in Eq. (15) is satisfied and if the time between two laser pulses is longer than the transient time in Eq. (16) then the laser pulse “projects” the atom onto states  $\rho_p^> \approx |1\rangle\langle 1|$  and  $\rho_p^0 \approx |2\rangle\langle 2|$ , respectively. This happens with nearly the same probabilities as predicted by projection postulate for an ideal measurement of the states  $|1\rangle$  or  $|2\rangle$ .

This will now be discussed in more detail. We assume the atom to be in an arbitrary initial state which may also have a  $|3\rangle$  component,

$$|\psi\rangle = \alpha_1|1\rangle + \alpha_2|2\rangle + \alpha_3|3\rangle . \quad (18)$$

The state of an atom which does not emit any photon while the laser pulse is on can be determined by using the conditional Hamiltonian and Eq. (13). The atom now practically evolves into the eigenstate of  $H_{\text{cond}}$  for the eigenvalue with smallest imaginary part. This will be denoted by  $|\lambda_2\rangle$ , and it has also a  $|3\rangle$  component. An elementary calculation gives

$$\lambda_2 = \frac{\Omega_2}{2} \epsilon_p (1 + \mathcal{O}(\epsilon^2)) \quad (19)$$

and

$$|\lambda_2\rangle = -i\epsilon_p|1\rangle + |2\rangle - \epsilon_R|3\rangle + \mathcal{O}(\epsilon^2) \quad (20)$$

$$|\lambda^2\rangle = i\epsilon_p|1\rangle + |2\rangle - \epsilon_R|3\rangle + \mathcal{O}(\epsilon^2) . \quad (21)$$

At the end of the laser pulse the state of an atom without emissions has thus evolved into

$$e^{-\lambda_2 \tau_p} \langle \lambda^2 | \psi \rangle \cdot |\lambda_2\rangle \quad (22)$$

and the probability for this is the norm squared,

$$P_0(\tau_p; \psi) = (1 - \epsilon_p \Omega_2 \tau_p) |\alpha_2|^2 + 2\epsilon_p \text{Im}(\alpha_1 \bar{\alpha}_2) - 2\epsilon_R \text{Re}(\alpha_2 \bar{\alpha}_3) + \mathcal{O}(\epsilon^2) \quad (23)$$

$$\approx |\alpha_2|^2 .$$

Thus at the end of a laser pulse the normalized state of an atom without fluorescence is given by

$$\rho^0(\tau_p) \equiv |\lambda_2\rangle \langle \lambda_2| = \begin{pmatrix} 0 & -i\epsilon_p & 0 \\ i\epsilon_p & 1 & -\epsilon_R \\ 0 & -\epsilon_R & 0 \end{pmatrix} + \mathcal{O}(\epsilon^2) . \quad (24)$$

To obtain the density matrix which describes an atom which does emit a burst of light during the laser pulse one has to average over all possible ways how photons can be emitted. This has been done for  $\Omega_3^2 \ll A_3^2$  in Ref. [18] and for the general case in Ref. [19]. Right at the end of a laser pulse an atom with emissions is shown to be in the (normalized) state

$$\rho^>(\tau_p) = \begin{pmatrix} A_3^2 + \Omega_3^2 & i\epsilon_p A_3^2 & iA_3 \Omega_3 \\ -i\epsilon_p A_3^2 & \epsilon_p \Omega_2 \tau_p A_3^2 & \epsilon_R (A_3^2 + \Omega_3^2) \\ -iA_3 \Omega_3 & \epsilon_R (A_3^2 + \Omega_3^2) & \Omega_3^2 \end{pmatrix} \times (A_3^2 + 2\Omega_3^2 + \epsilon_p \Omega_2 \tau_p A_3^2)^{-1} + \mathcal{O}(\epsilon^2) \quad (25)$$

which has non-negligible  $|3\rangle$  components and which is also independent from the initial state  $|\psi\rangle$  of the atom. (Strictly speaking, Eq. (25) holds only if the leading contribution  $1 - |\alpha_2|^2$  of  $1 - P_0(\tau_p; \psi)$ , the probability to detect photons, does not vanish or becomes itself  $\mathcal{O}(\epsilon)$ . In these exceptional cases  $\rho^>$  *does* depend on the initial state  $|\psi\rangle$ .)

After the end of the laser pulse the  $|3\rangle$  components decay during the transient time. Simultaneously the 1–2 transition is weakly pumped. For times long enough for the third level contributions to have vanished it has been shown in Ref. [19] that a time  $\tau$  after the end of the laser pulse an atom without any emission is in the state

$$\rho^0(\tau_p + \tau) = U(\tau) \rho_p^0 U^\dagger(\tau) , \quad (26)$$

where  $U(\tau)$  is the “free” time development operator of Eq. (4), which describes the small driving of the 2-level atom by the weak field and where  $\rho_p^0$  is in the 1–2 subspace and given by

$$\rho_p^0 = \begin{pmatrix} 0 & -i\epsilon_p \\ i\epsilon_p & 1 \end{pmatrix} + \mathcal{O}(\epsilon^2) \approx |2\rangle \langle 2| . \quad (27)$$

Analogously an atom with emissions is, a time  $\tau$  after the end of the laser pulse, in the (normalized) state

$$\rho^>(\tau_p + \tau) = U(\tau) \rho_p^> U^\dagger(\tau) \quad (28)$$

with

$$\begin{aligned} \rho_p^> &= \begin{pmatrix} A_3^2 + 2\Omega_3^2 & i\epsilon_p A_3^2 - \frac{i}{2}\epsilon_A \Omega_3^2 \\ -i\epsilon_p A_3^2 + \frac{i}{2}\epsilon_A \Omega_3^2 & \epsilon_p \Omega_2 \tau_p A_3^2 \end{pmatrix} \\ &\times (A_3^2 + 2\Omega_3^2 + \epsilon_p \Omega_2 \tau_p A_3^2)^{-1} + \mathcal{O}(\epsilon^2) \approx |1\rangle\langle 1|. \end{aligned} \quad (29)$$

Eqs. (26) and (28) suggest the interpretation that, under the stated conditions, the laser pulse effectively “projects” the atom at time  $\tau_p$  onto the states  $\rho_p^0$  or  $\rho_p^>$ , respectively, which then undergo a “free” time development. The probabilities to find the atom in  $\rho_p^0$  or  $\rho_p^>$  are nearly the same as for an ideal measurement of the states  $|1\rangle$  or  $|2\rangle$ . However, in particular in this case the laser pulse is not quite an ideal measurement, and corrections have been given in the above formulas.

#### 4. Ensembles: Discussion of the experiment of Itano et al.

Now the experiment of Itano et al. [7] to test the quantum Zeno effect on an ensemble of atoms can be analyzed in more detail. As shown in Fig. 6 and discussed in the Introduction they had an ensemble of atoms in a trap (a gas with negligible cooperative effects) and applied a weak field for the duration  $T_\pi = \pi/\Omega_2$  (a  $\pi$  pulse). During this  $\pi$  pulse  $n$  strong laser pulses of duration  $\tau_p$  were applied. Initially all atoms were prepared in the ground state. Therefore without the strong laser pulses all atoms would be in the state  $|2\rangle$  at the end of the  $\pi$  pulse.

Every single atom is influenced by the strong laser pulses. Therefore, the effect of every laser pulse on the ensemble can be regarded as simultaneous and approximately ideal measurements on each single atom (with corrections, as discussed in Section 3).

Itano et al. determined experimentally the population of level 2 at the end of the  $\pi$  pulse for  $n$  laser pulses during this time [7]. Their results are shown in the last column of Table 1. If one interprets the effect of a laser pulse as an ideal and instantaneous measurement one obtains the first column. Better results are obtained by assuming an ideal state reduction and taking the finite duration of the laser pulse for its realization into account (second column).

The results of the last section have been used elsewhere [18, 19] to analytically calculate the population of level 2, i.e. with the proper corrections up to order  $\epsilon^2$  to the projection postulate taken into account. With the parameters of the experiment we obtain the third column of Table 1. As seen in column 4, a numerical solution of the corresponding Bloch equation leads to comparable results, although it does not give the same direct physical insight.

| $n$ | Projection Postulate |                               | Quantum<br>Jump | Bloch<br>equations | Observed<br>[7] |
|-----|----------------------|-------------------------------|-----------------|--------------------|-----------------|
|     | $\Delta t = T_\pi/n$ | $\Delta t = T_\pi/n - \tau_p$ |                 |                    |                 |
| 1   | 1.00000              | 0.99978                       | 0.99978         | 0.99978            | 0.995           |
| 2   | 0.50000              | 0.49957                       | 0.49960         | 0.49960            | 0.500           |
| 4   | 0.37500              | 0.35985                       | 0.36062         | 0.36056            | 0.335           |
| 8   | 0.23460              | 0.20857                       | 0.20998         | 0.20993            | 0.194           |
| 16  | 0.13343              | 0.10029                       | 0.10215         | 0.10212            | 0.103           |
| 32  | 0.07156              | 0.03642                       | 0.03841         | 0.03840            | 0.013           |
| 64  | 0.00371              | 0.00613                       | 0.00789         | 0.00789            | -0.006          |

Table 1: Predicted and observed population of level 2 at the end of the  $\pi$  pulse for  $n$  laser pulses of length  $\tau_p$ .  $\Delta t$  is the time between two measurements.

## 5. A possible experiment on a single atom

It should be possible to perform the following experiment on a single atom in a trap. The weak field which drives the 1–2 transition, is kept continuously on and will not be turned off after time  $T_\pi$ . In addition, strong laser pulses of length  $\tau_p$  are applied repeatedly at times  $\Delta t$  apart, as discussed in Section 3 and depicted in Fig. 7.

A measurement point of view gives a quick and intuitive understanding what to expect, namely a stochastic sequence of fluorescence bursts forming light periods alternating with dark periods. Their duration should increase with decreasing distance between the laser pulses.

On the other hand, a dynamical point of view can directly employ the results of Section 3. With those results one can determine the probability to find no emissions during a laser pulse, if there had been a burst of fluorescence or no photons, respectively, during the preceding laser pulse. Eq. (5) is now replaced by

$$\begin{aligned} P(\text{emissions} \rightarrow \text{no emissions}) &= P(\rho_P^> \rightarrow \rho_P^0) \equiv p, \\ P(\text{no emissions} \rightarrow \text{no emissions}) &= P(\rho_P^0 \rightarrow \rho_P^0) \equiv q, \end{aligned}$$

provided that  $\Delta t$  is not too short,

$$\Delta t \gg 1/A_3 \quad \text{and} \quad (\Omega_2 \Delta t)^2 \gg \epsilon. \quad (30)$$

The first of these conditions ensures that the  $|3\rangle$  components of  $\rho^0$  and  $\rho^>$  have enough time to decay completely so that it is possible to make use of the states  $\rho_P^0$  and  $\rho_P^>$ . If the second condition is violated then the state at the beginning of the *first* pulse in a light period is very close to  $\rho^0$ , and therefore the state  $\rho^>$  after the first pulse has to be calculated with initial state of the form  $\rho^0 + \mathcal{O}(\epsilon)$ .

For such a state, however, one has  $1 - P_0 = \mathcal{O}(\epsilon)$  so that Eq. (25) fails. Thus, if the second condition in Eq. (30) does not hold the first pulse in a light period has to be treated differently from the rest.

The detailed calculation is given elsewhere [29]. Under the above conditions the result is

$$p = \sin^2 \frac{\Omega_2}{2} \Delta t + \epsilon_p \left( 2s \frac{A_3^2 + \Omega_3^2}{A_3^2 + 2\Omega_3^2} + \frac{1}{2} \Omega_2 \tau_p c \frac{3A_3^2 + 2\Omega_3^2}{A_3^2 + 2\Omega_3^2} - \frac{1}{2} \Omega_2 \tau_p \right) - \frac{1}{2} \epsilon_A s \frac{\Omega_3^2}{A_3^2 + 2\Omega_3^2} + \mathcal{O}(\epsilon^2) \quad (31)$$

$$q = \cos^2 \frac{\Omega_2}{2} \Delta t - \epsilon_p \left( 2s + \frac{1}{2} \Omega_2 \tau_p (1 + c) \right) + \mathcal{O}(\epsilon^2) \quad (32)$$

with  $s \equiv \sin \Omega_2 \Delta t$  and  $c \equiv \cos \Omega_2 \Delta t$ .

The probability for a period of exactly  $n$  consecutive laser pulses *with* fluorescence among all such light periods is  $(1 - p)^{n-1} p$ . The mean duration  $\bar{T}_L$  of light periods is therefore

$$\bar{T}_L = \sum_{n=1}^{\infty} (\tau_p + \Delta t) n (1 - p)^{n-1} p$$

which gives

$$\bar{T}_L = \frac{\tau_p + \Delta t}{p} . \quad (33)$$

Similarly one finds for the dark periods

$$\bar{T}_D = \frac{\tau_p + \Delta t}{1 - q} . \quad (34)$$

These results are now a little bit different from the case of ideal measurements as discussed in Section 2. Since  $1 - q$  is close, but not equal, to  $p$  there is now a small asymmetry between light and dark periods.

In spite of the problems arising for  $\Delta t \rightarrow 0$  this limit can be performed. The result is (details can again be found in [29])

$$\lim_{\Delta t \rightarrow 0} \bar{T}_D = \frac{\Omega_3^2}{\Omega_2^2 A_3} , \quad \lim_{\Delta t \rightarrow 0} \bar{T}_L = \frac{\Omega_3^2 (A_3^2 + 2\Omega_3^2)}{\Omega_2^2 A_3^3}$$

up to terms of relative order  $\epsilon/\Omega_2 \tau_p$ . In contrast to Eq. (8) for ideal measurements  $\bar{T}_D$  and  $\bar{T}_L$  remain finite, as physically expected. For  $\Delta t = 0$  both driving fields are continuously on. In this case the existence of light and dark periods is well known under the name “electron shelving” [30], for which the same results for  $\bar{T}_D$  and  $\bar{T}_L$  have been obtained [31].

## 6. Conclusions

The predictions of the quantum Zeno effect for a single system and an ensemble under rapidly repeated measurements have been studied for the example

of state measurements on a two-level atom. To test the quantum Zeno effect an experiment on an ensemble of atoms was performed by Itano et al. in which an atomic level measurement was realized by means of a short laser pulse.

An explanation for the approximately allowed applicability of the projection postulate to this case has been given by us using the quantum jump approach. We have determined corrections to the ideal case explicitly. We have used these results to discuss the experiment of Itano et al. [7] and a new possible experiment with a single atom in some detail. The projection postulate has been found to be an excellent pragmatic tool for quick and fairly accurate answers and for a simple intuitive understanding. However, corrections to it arise.

In the Introduction we have mentioned a controversy regarding the quantum Zeno effect in general and the role of the experiment of Ref. [7] in particular. We think that our analysis sheds some light on this. It is, in our opinion, perfectly legitimate to take a ‘puristic’ view that for example the laser pulses (“measuring pulses”) have nothing to do with measurements but just lead to additional terms in the Hamiltonian. Then any change in the temporal development is not surprising and may in principle be calculated with these additional interaction terms. However, the actual temporal behavior is in general not easily seen and will often need numerical evaluations which may give little physical insight. The other, more fruitful, point of view is that these laser pulses *approximately* realize measurements with state reductions. Then one immediately has simple predictions for the approximate behavior of the system and understands the slow-down of the time evolution without complicated calculation. Finer details require of course a finer analysis. An actual freezing of the state does not seem possible since all realistic measurements take a finite time. In the present case this is explicitly seen in the finite duration of the laser pulse and the required minimal time between them.

In a broader sense our analysis also sheds some light on the use of the projection postulate in general, not only in connection with the Zeno effect. It seems that quite often the projection postulate is a useful tool which can give quick and fairly accurate answers. The accuracy depends on how far the particular realistic measurement differs from an ideal measurement as considered in orthodox quantum mechanics, and corrections may have to be taken into account. An idealization of realistic measurements and the projection postulate may often be very useful. Over-idealizations, however, are dangerous since they can lead to interpretational difficulties and to ‘paradoxes’ like the freezing of states in the limit of ‘continuous ideal measurements’. In this limit the idealization becomes an overidealization and breaks down.

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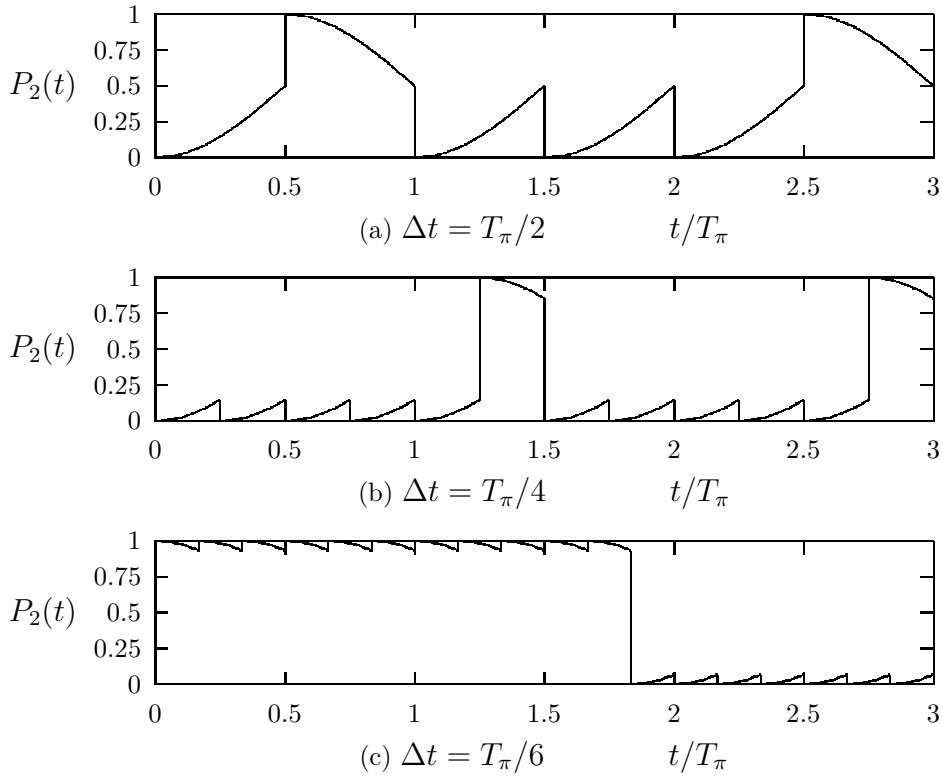


Figure 1: Possible paths of the population  $P_2(t)$  of level 2 of a single atom. Ideal measurements at times  $\Delta t$  apart project the atom on state  $|1\rangle$  or  $|2\rangle$  respectively. The time is given in multiples of the length of a  $\pi$  pulse  $T_\pi$  with  $T_\pi = \pi/\Omega_2$ .

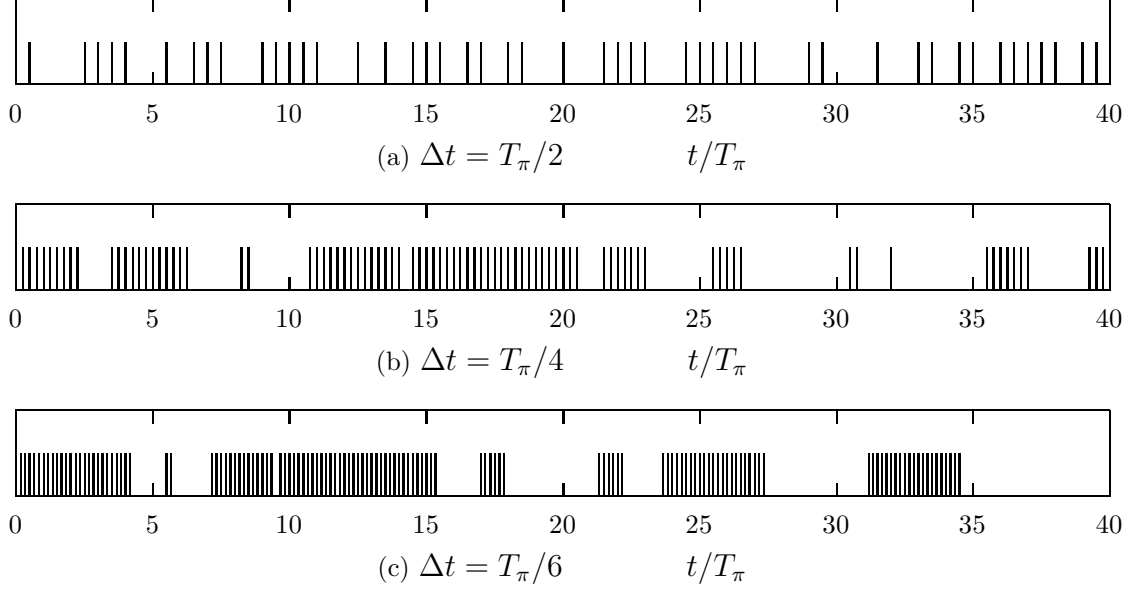


Figure 2: Stochastic alternating light and dark periods. The lines mark times when the atom is found in state  $|1\rangle$ , with accompanying light signal.  $T_\pi = \pi/\Omega_2$  is the duration of a  $\pi$  pulse.

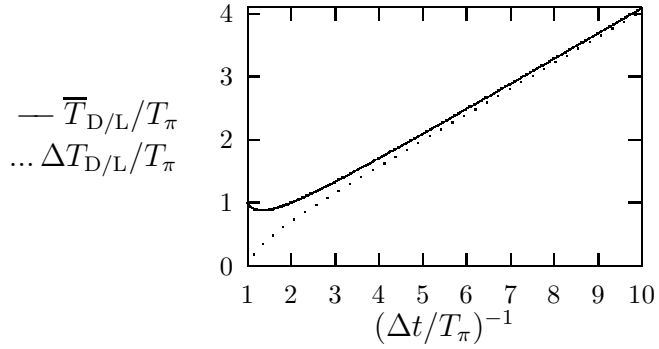


Figure 3: Mean length  $\overline{T}_{D/L}$  and standard deviation (dotted line) of light (dark) periods as a function of the inverse of the time between two measurements  $\Delta t$ .  $T_\pi = \pi/\Omega_2$  is the duration of a  $\pi$  pulse.

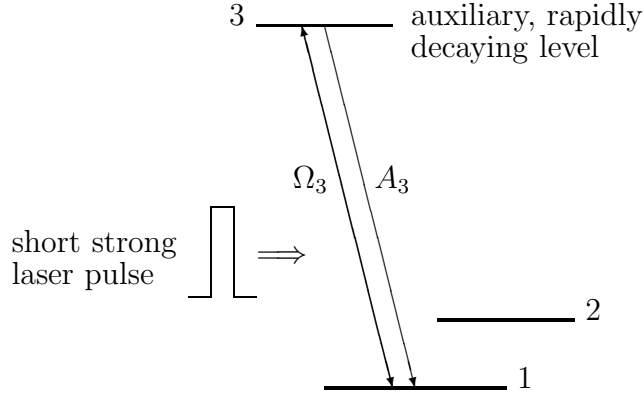


Figure 4: V system with (meta)stable level 2 and Einstein coefficient  $A_3$  for level 3.  $\Omega_3$  is the Rabi frequency of the short strong laser pulse of duration  $\tau_p$ . The 1–2 transition is not driven here.

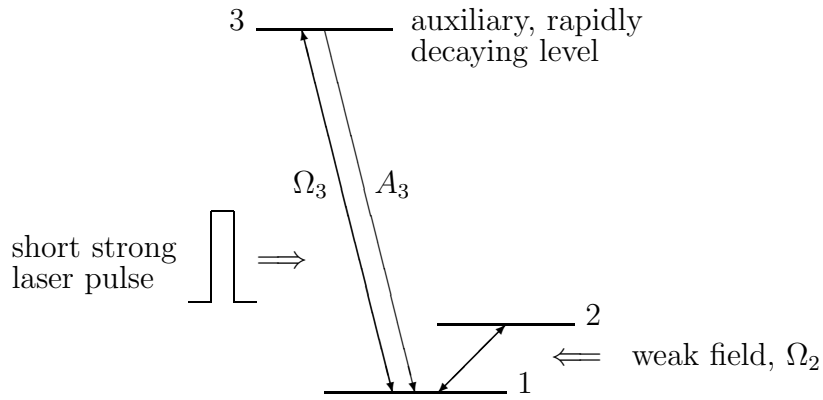


Figure 5: V system as in Fig. 4 with an additional weak field (Rabi frequency  $\Omega_2$ ).

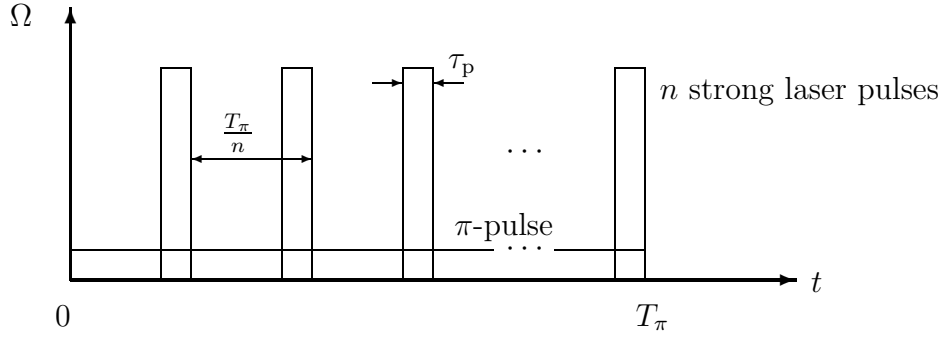


Figure 6: Strong laser pulses and  $\pi$  pulse as applied in the experiment of Itano et al. Initially all atoms are prepared in the ground state.  $T_\pi$  is the duration of a  $\pi$  pulse.

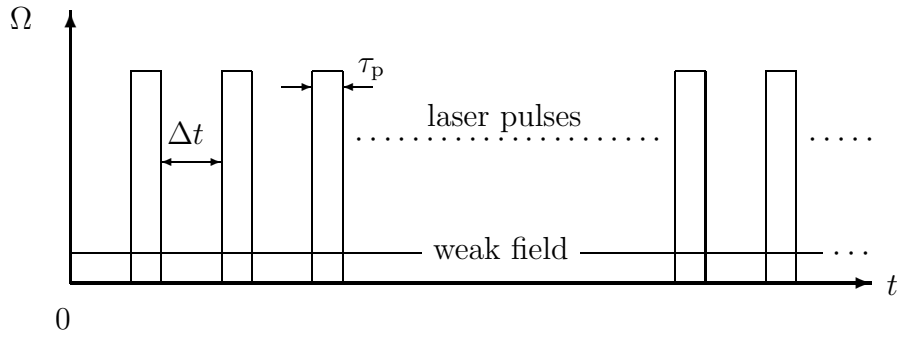


Figure 7: Proposed experiment on a single atom. The weak field driving the 1–2 transition is kept on continuously. At times  $\Delta t$  apart strong laser pulses are applied.